Joanna Major

Tangent to the graph of the function - nonstandard approach

Geometrically, the derivative is interpreted as follows. Let a curve $y = f(x)$ be given and let us draw a straight line through the points $[a , f(a)]$ and $[a + h , f(a + h)]$, h being a fixed positive value. This straight line is called a secant with regard to the given curve. It is easily seen that the difference quotient $g(h)$ is the tangent of the angle a_h between the secant directed by the increasing abscissae and the x-axis. The limiting position to which the secant tends as h tends to 0 will be considered as the position of the tangent (Kuratowski K., Introduction to Calculus, 1962). But how do we show this {it limiting position}? In our talk we sketch the basic facts of nonstandard analysis, i.e. in the product $mathbb{R}^mathbb{N}$ we define an equivalence relation on $mathbb{N}$ by using $(a_n)equiv{(b_n)}Leftrightarrow_{df}{nin{mathbb{N}}:a_n=b_n} in{mathcal{F}}.$ The set of hyperreal $mathbb{R}^*$ is the quotient set $mathbb{R}^mathbb{N}/_{equiv}$. Algebraic operations on $mathbb{R}^*$ are defined in a standard way. The structure $mathfrak R^*=(mathbb{R}^{ast}, +, cdot,1^ast, 0^ast, prec)$ Is a totally ordered, non-Archimedean field (see Goldblatt R., Lectures on the Hyperreals, An Introduction to Nonstandard Analysis, 1998). In the last part of our talk we sketch to get the limiting position of the secant (tangent to the graph of the function) in the structure $mathfrak R^*$ without using limits.

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