Pedagogical University of Cracow 15 – 19 March 2015

II INTERDISCIPLINARY SCIENTIFIC CONFERENCE

MATHEMATICAL TRANSGRESSIONS

BOOK OF ABSTRACTS

Editors
P. Błaszczyk & J. Major

TR mathematical TRANSGRESSIONS

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Keynote Speakers

Alan Schoenfeld
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Charles University in Prague, Czech Republic

Shlomo Vinner
Hebrew University of Jerusalem; Ben-Gurion University of the Negev, Israel

Kobus Maree
University of Pretoria, South Africa

"Mathematical Transgressions" Conference - General Schedule

Sunday, 15 March	Monday, 16 March	Tuesday, 17 March	Wednesday, 18 March	Thursday, 19 March
	8:30-9:00 - Auditorium Opening Ceremony	translation equipment set up break	translation equipment set up break	translation equipment set up break
	translation equipment set up break	9:00-10:00 Invited talk: Erich Ch. Wittmann Auditorium	9:00-10:00 Invited talk: Bernard Sarrazy Auditorium	9:00-10:00 Invited talk: Shlomo Vinner Auditorium
	9:15-10:15 Invited talk: Alan Schoenfeld Auditorium	coffee break - Hall	coffee break - Hall	coffee break - Hall
	coffee break - Hall	10:30-11:30 Invited talk: Marianna Ciosek, Stefan Turnau Auditorium	10:30-11:30 Invited talk: Jarmila Novotna Auditorium	10:30-11:30 Invited talk: Jacobus Maree Auditorium
10:00-13:15 Registration Pedagogical University Podchorążych 2 Hall	10:45-11:45 Invited talk: Paul Ernest Auditorium			
		11:45-13:30 Plenary session 2 Auditorium	11:45-13:30 Plenary session 3 Auditorium	11:45-13:30 Plenary session 4 Auditorium
	12:00-13:45 Plenary session 1 Auditorium			
		13:30-14:15 Lunch Hall	13:30-14:15 Lunch Hall	13:30-14:00 - Auditorium Closing Ceremony
	13:45-14:30 Lunch Hall			14:00-14:45 Lunch Hall
13:15-17:00 Tour of Krakow Meeting Point - in front of Pedagogical University Podchorążych 2		14:30-16:00 Session 2A/Session 2B Auditorium/Aula	14:30-16:00 Session 3A/Session 3B Auditorium/Aula	
	14:45-16:15 Session 1A/Session 1B Auditorium/Aula	coffee break - Hall	coffee break - Hall	
	coffee break - Hall	16:30-18:00 Workshop: Alan Schoenfeld Auditorium	16:30-17:30 Workshop: Istvan Lenart Auditorium	
	16:45-18:15 Workshops: 1. GWO - Marcin Karpiński 2. Jacobus Maree Aula/Auditorium	18:00-18:30 Vernissage: Jakub Jernajczyk, MathArt	17:30-18:30 Meeting with invited speakers Aula	
17:15-18:30 Registration Pedagogical University Podchorążych 2 Hall	18:15-19:00 (in Polish) Workshop (Aula) GWO - B. Pieronkiewicz M. Samborska, M. Kubat			
		20:00 Conference Dinner		

About *transgressio*

It has been almost 70 years since the Pedagogical University of Cracow, the host of this conference, was established. It was founded on May 11, 1946, originally as the National Higher College of Teacher Training. Since its very beginning, the Pedagogical University aims to train highly qualified teaching staff for the Polish educational system. University-led training of mathematics teachers in Poland was dominated by the research activity of Prof. Zofia Krygowska at the Pedagogical University. All through her academic career, from the early 50s to the late 80s of the past century, she has been developing the Polish school of didactics of mathematics.

As the last decades of the 20th century have passed, we have entered the Digital Era. The Third Technological Revolution has an enormous socio-economic impact; what is of particular importance is that, it affects mathematics itself, and it also strives to change the entire education system. Mathematicians seek to focus on the, so-called, concrete mathematics, they explore finite and discrete structures, rather than infinite and continuous ones, they prefer to develop combinatorics and algorithmic thinking, rather than contribute to Bourbaki's edifice; mathematics expands its boundaries, merging with computer science research. We have to concede that mathematics is no longer "the Queen of the Sciences"; however, mathematics is still believed to be the basis of modern education, although its role needs to be re-defined. We have to address this challenge.

The Latin word *transgressio* means an act that goes beyond generally accepted boundaries. We believe that a comprehensive understanding of students' school situation and an effective support of school training needs diverse perspectives. That is why we have invited teachers of mathematics, mathematicians, pedagogues, psychologists and philosophers to share their knowledge and expertise at this conference.

Invited Talks

Invited Talk 1 (Monday, 16 March, 9:15–10:15)

Alan Schoenfeld (University of California, USA)

What makes for powerful classrooms, and how can we support teachers in creating them?

Invited Talk 2 (Monday, 16 March, 10:45–11:45)

Paul Ernest (Exeter University, United Kingdom)

Challenging Myths about Mathematics

Invited Talk 3 (Tuesday, 17 March, 9:00–10:00)

Erich Ch. Wittmann (Technical University of Dortmund, Germany)

Structure-genetic didactical analyses – empirical research of the first kind

Invited Talk 4 (Tuesday, 17 March, 10:30–11:30)

Marianna Ciosek (Pedagogical University of Cracow [retired], Poland),

Stefan Turnau (University of Rzeszów [retired], Poland)

Harmonizing different kinds of mathematical thinking in mathematics education

Invited Talk 5 (Wednesday, 18 March, 9:00–10:00)

Bernard Sarrazy (Université de Bordeaux, France)

Contract, transgressions and creation. An attempt to clarify the paradoxes of the didactical relationship in mathematics education using a didactical and anthropological approach

Invited Talk 6 (Wednesday, 18 March, 10:30–11:30)

Jarmila Novotná (Charles University in Prague, Czech Republic)

Developing a Culture of problem solving through heuristic strategies

Invited Talk 7 (Thursday, 19 March, 9:00–10:00)

Shlomo Vinner (Hebrew University of Jerusalem; Ben-Gurion University of the Negev, Israel)

Scientific thinking versus religious thinking from a view point of a secular science educator

Invited Talk 8 (Thursday, 19 March, 10:30–11:30)

Jacobus Maree (University of Pretoria, South Africa)

Career Construction in the mathematics classroom: Using an integrated, qualitative+quantitative approach to enhance

Plenary Sessions

Plenary Session 1 (Monday, 16 March, 12:00–13:45)

Chair: Piotr Błaszczyk

- Ewa Swoboda

  "Transgressions" of a didactical problem concerning geometric reasoning. From the researcher's workshop

- Jeppe Skott

  The Goldilocks Principle revisited: Balancing "proving that" and "proving why" in mathematics teacher education

- Paola Vighi

  Abstract paintings, objects and actions: how to promote geometrical understanding

- Anita Sondore, Elfrīda Krastina

  The comprehension of elements of combinatorics in real-life situations for primary school students

Plenary Session 2 (Tuesday, 17 March, 11:45–13:30)

Chair: Mirosława Sajka

- Maciej M. Sysło

  Learning Mathematics Supported by Computational Thinking

- Bohdan Józef Naumienko

  The Mathematical Foundations of Integral Education

- Jerzy Pogonowski

  Paradox resolution as a didactic tool

- Waclaw Zawadowski

  How to use a pocket calculator for developing and sustaining readiness in mental calculation

Plenary Session 3 (Wednesday, 18 March, 11:45–13:30)

Chair: Tomasz Szemberg

- Ralf Benölken

  Research results on mathematical talent, gender and motivation

- Hyung Kim

  Developing mathematics teachers' pedagogical identity in the classroom context

- Valéria Csépe

  Neurocognitive development of magnitude processing

- Hannes Stoppel

  Development of students' beliefs in mathematical understanding in relation to mathematics and its applications

Plenary Session 4 (Thursday, 19 March, 11:45–13:30)

Chair: Piotr Błaszczyk

- Manuela Moscucci

  About the relation between relationships and teaching and learning mathematics

- Jerome Proulx

  Looking at students: from a medical/deficit view on mathematical knowledge toward possibilities of mathematical actions

- András Ambrus

  Applying Cognitive Load Theory in mathematics education

- Miroslawa Sajka

  Eyetracking as a new method for research on mathematics education

Parallel Sessions

Session 1A (Monday, 16 March, 14:45–16:15)

Chair: Jeppe Skott

- Hubert Bożek

  Logic as an Anti-platonic Antidtote. A Case from Chwistek

- Magdalena Kubat

  An unusual polyhedron in students' mathematical education

- Paweł Perekietka

  The contributions of computer science to mathematical education. Examples from outside curriculum

- Barbara Pieronkiewicz

  Affective transgression in learning mathematics

Session 1B (Monday, 16 March, 14:45–16:15)

Chair: Ewa Swoboda

- Jakub Jernajczyk

  The circle – mathematical inspirations in philosophy and art

- Agata Hoffmann

  Stanisław Dróżdż and mathematics

- Michalina Kasprzak

  Teaching mathematics creatively through theatre for preschool children and in early childhood education

- Barbara Wawrzacz

  The use of building blocks in construction of squares and cubes representing the numbers – transition from the concrete world to the world of operational reasoning

Session 2A (Tuesday, 17 March, 14:30–16:00)

Chair: Jacek Chmieliński

- Hyung Kim

  Students' understanding of expected value of a random variable

- Yuuki Mori

  Computational Complexity of College Math Eigenvalue Problems

- Mette Andresen

  Students' strategies for problem solving in upper secondary

Session 2B (Tuesday, 17 March, 14:30–16:00)

Chair: Mirosława Sajka

- Alina Kalinowska

  Day to day of creating mathematical concepts in early education

- Gabriela Biel

  Tutoring as a method of mathematics education

- Maria Samborska

  Engaging underachieving students in mathematical thinking. An example of a supplementary math course for middle school students

- Agnieszka Proszewska

  The Unreasonable Effectiveness of Mathematics or the Simple Efficiency of Linguistic Model?

Session 3A (Wednesday, 18 March, 14:30–16:00)

Chair: Piotr Błaszczyk

- Eva Nováková

  Prediction and self-evaluation of pupils in solving non-standard mathematical tasks

- Anna Baczko-Dombi

  In the Search for Reasons Behind Students Escaping from Mathematics

- Maria Gokieli, Marcin Szpak
  Experiment and visualization in school mathematics
- Anna Kucharzewska, Artur Nyklewicz
  Does a mathematician need a university?

Session 3B (Wednesday, 18 March, 14:30–16:00)

Chair: Maciej M. Sysło

- Volha Malinouskaya, Urszula Oszwa
  Young mathematical talents in Belorussian educational system
- Lidia Zaręba
  The role of visualization in the process of generalization
- Marek Janasz
  Automated theorem proving for Euclidean geometry
- Joanna Major
  On graphic representation of a tangent line to a function

Workshops

Workshop I (Monday, 16 March, 16:45–18:15)

- Marcin Karpiński

  How not to lose them? About students' problems with mathematics Conclusions from IBE research and practical guidelines for daily work in the classroom

A considerable number of students entering 4th grade experience problems with mathematics. Problems with understanding particular issues accumulate at different stages of learning and reveal themselves even around the matriculation exam. Mastering this subject causes a lot of trouble to weak students, and more capable as well.

During his course, the Author is going to indicate those skills that students find most difficult to master. You will also be given some hints on how not to "lose" students with shortcomings rooted in their former years of mathematical education.

Workshop II (Monday, 16 March, 16:45–18:15)

- Jacobus Gideon Maree

  How to get published in high-impact scholarly journals: The art, science and skill of article writing

The aim of this workshop is to help emerging scholars get published in high-impact scholarly journals. Participants should clarify their reasons for wanting to publish – for instance, attaining scholar status, pushing the boundaries of knowledge, testing the water, or challenging certain conventions, and obtain clarity as to why they do not (want to) publish. In addition, attendees will be alerted to the different kinds of academic writing and guided through the basic steps in preparing manuscripts for submission, including how to write scholarly articles (introduced to the basic scientific organization of scholarly articles), where and how to start writing for the journal, how to choose a journal for submission, and testing drafts in scholarly forums, such as conferences and seminars. Moreover, attendees will discover why language editing is so important, what reviewers are looking for in manuscripts, how they should go about contacting editors and how to deal with reviewers' comments. It is hoped that all attendees will publish at least one article in a high-impact scholarly journal within 12 months of the workshop.

Workshop III (Monday, 16 March, 18:15–19:00)

- Magdalena Kubat, Barbara Pieronkiewicz, Maria Samborska

  Developing mathematical thinking – task solving

We will present some mathematical tasks which can enhance students' activity. On the basis of our teaching practice, we surmise that by using these tasks we can also draw out students considered as "weak" or "capable, but lazy". We invite all attendees to join the workshop and solve some tasks together! Let this workshop be a sample of classes that everyone would like to take part in.

Workshop IV (Tuesday, 17 March, 16:30–18:00)

- Alan Schoenfeld

  Looking at Classrooms: The Teaching for Robust Understanding (TRU) Framework

In my plenary presentation I outlined a framework for thinking about the aspects of powerful mathematical classrooms. But we didn't have the time to look at examples of classroom instruction. Here I would like to look at real examples of classrooms, talk about what happens in them, and see whether the Teaching for Robust Understanding (TRU) framework helps us understand what happens in them.

Workshop V (Wednesday, 18 March, 16:30–17:30)

- Istvan Lenart

  Mathematics in statu nascendi as a Tool for Didactics in statu nascendi

Didactics of mathematics, in its state of birth as a scientific discipline, formulates its rights and its duties. One of the main challenges is to raise the students' interest for the magnificent mixture of philosophy, natural sciences, poetry and art that is commonly called Mathematics. We teachers/educators should convey the picture of a subject open to constant change and improvement, endowed with the beauty of a living body, in contrast with "the cold and austere beauty" of a sculpture. To achieve our goal, this talk/ workshop tries to highlight "AHA" moments in selected mathematical topics when, in a flash, a mathematical idea becomes clear for the researcher or for the student. As Freudenthal put

it, "The opposite of ready – made mathematics is mathematics in statu nascendi. This is what Socrates taught. Today we urge that it be a real birth instead of a stylized one; the pupil himself should re-invent mathematics." We hold his statement to be true both inside and outside of mathematics. The way of teaching to follow may be more complex and time-consuming for the practising teacher than one of simply "filling up the vessel"; but it pays the daily effort back by fostering the curiosity and creativity in our students with very diverse backgrounds regarding their ability, gender, social status or future profession.

Vernissage (Tuesday, 17 March, 18:00–18:30)

- Jakub Jernajczyk

  MathArt

Exhibition of artworks inspired by issues of mathematics, such as non-Euclidean geometries, irrational numbers, dichotomous divisions, kinematic curves, infinity.

ABSTRACTS

by alphabetical order of the speakers

András Ambrus

Eötvös Lóránd University Budapest, HU

Applying Cognitive Load Theory in mathematics education

Abstract. To characterize the quality of teaching mathematics in a country is not an easy task. Outside of Hungary you may hear very often: "The Hungarian way of teaching mathematical problem solving world-famous thanks to Georg Pólya." Maybe they know of some excellent mathematicians: J. Neumann, P. Halmos, E. Szemerédi, L. Lovász. It is true, but does not say anything about the real situation of Hungarian mathematics teaching today. To characterize it, we will analyze, in details the PISA 2012 mathematics test results and one university mathematics test, taken at the start of higher education studies. Further basing on these studies, we narrow our attention to students who continue their education at universities and, colleges. This is the top 20-25% of a year. The article tries to answer a question: How can we prepare these students more effectively for their studies and, for their future jobs? Our statement is: use of open problems is taking one step forward to help more students be more successful in mathematical problem-solving. After some theoretical consideration, we analyze the tests of different students, as well as their ideas, and make some suggestions on how to take steps forward to reaching more students in Hungary to teach mathematical problem solving. Finally, we summarize some suggestions about how to spread the research results into the mainstream of teaching Hungarian mathematical problem solving.

Mette Andresen

University of Bergen, NO

Students' strategies for problem solving in upper secondary mathematics classes

Abstract. This presentation reports the first results of a research project focusing on the development of the students' inquiry, creativity and intellectual independence when working in a problem solving setting in upper seconary mathematics classes. Eight mathematics teachers prepared and conducted teaching experiments for our observation of the new strategies gradually developed by the students. The theoretical basis of the project includes works of Schoenfeld, Polya and Cobb et al. and Johan Lithner.

Anna Baczko-Dombi

POLISH ACADEMY OF SCIENCES, PL

In the Search for Reasons Behind Students Escaping from Mathematics

Abstract. Mathematics, unlike any other school subject, evokes conflicting emotions and contradictory attitudes – from the "Queen of Sciences" to widespread acceptance for mathematical ignorance. The reaction to mathematics in society seems to be based on stereotypes that "mathematicians" are opposed to "humanists". The process of studying mathematics requires systematic practice and patience, as mathematical knowledge is of cumulative nature. The promise of awards, such as a good job in the future, and the threat of the obligatory matriculation exam turn out to be insufficient.

In the case of mathematical education we can observe a process of abandoning mathematics by a part of the students who start to consider themselves "humanists". It causes serious decisions about their profile of education – e.g. class profile. This decision has numerous consequences, as it may narrow down possible paths of future education and career, finally resulting in employment and income inequalities. This phenomenon can be named "mathematical exclusion".

In my presentation I intend to show the most important elements of the image of mathematicsand compare how they are regarded by students, teachers and parents. I intend to show the sources of the abandonment of mathematics in the negative attitudes towards mathematics and a specific perception of the usability of mathematical knowledge in a short and long time-perspective. I will refer to selected results of two studies on Polish students – qualitative (2010), and quantitative (2011, survey on students ($N = 3169$), also including a sample of teachersand parents).

Ralf Benölken

University of Muenster, DE

Research results on mathematical talent, gender and motivation

Abstract. Problem statement: In Germany, girls are decidedly underrepresented in programs that foster mathematical talent at primary school age. Thus, it is of interest to ascertain aspects of improving their identification and support.

Purpose: Two studies were conducted to clarify the significance of motivational factors as determinants for the identification of talent by comparing girls and boys who were identified to be mathematically talented (imt) as well as girls and boys who were not (n-imt). The first study focused on self-concepts, attributions and general interests, the second on attitudes and mathematics interest.

Method: Children of the 3rd and 4th grade were asked using standardized questionnaires. The first study's sample covers $N = 288$ (132f, 156m), including $n = 165$ imt children (66f, 99m); the second one's $N = 162$ (71f, 91m), including $n = 83$ imt children (32f, 51m). The data have been evaluated by an analysis of variance with the factors being talent and sex.

Results: The characteristics of all examined motivational factors were more adventageous with imt children and n-imt boys than with n-imt girls.

Conclusion: Disadvantageous motivational factors seem to be important aspects of explaining the infrequent identification of girls' talent. Boys' talents might be identified more often because – independent of talent-identification – they might tend to have a strong preoccupation with mathematics, and teachers might perceive their talent more easily. Future research should focus on the significance of motivational factors as determinants for talent-development.

Practical consequences: Any gender-stereotyping of mathematics should be avoided. It seems to be important to develop girls' motivational factors consciously to support their talents.

Gabriela Biel

UNIWERSYTET PRZYRODNICZY WE WROCŁAWIU, PL

Tutoring as a method of mathematics education

Abstract. Attempts of explaining the word 'tutoring' in Polish leads to many definitions. One of them talks about a "planned development process which takes place in the individual, supportive relationship with another person – tutor" [1]. Two main goals of 'tutoring' treat about extracting fully the participant's potential as well as self-reliance in self-development and self-education. It is also important to see this kind of teaching as an opportunity for a revival of the mission of Polish universities [2]. Since the openness and commonness of universitiesis becoming a reality – it is desirable to create new paths for students who want to learn more and more deeply [3]. Tested in a high school environment, math tutorials are likely to deepen the mathematical skills of the students of courses in which mathematical education is limited to a minimum. This paper will present the main objectives of school and academic 'tutoring' in relation to mathematics teaching on the basis of theoretical studies and of own experiences.

[1.] K.Czayka-Chełmińska, *Metoda tutoringu*, w: Polska-Amerykańska Fundacja Wolności (Liderzy PAFW), W poszukiwaniu metody kształcenia liderów, Warszawa 2007, Stowarzyszenie Szkoła Liderów, s. 96.

[2.] P. Czekierda, *Tutoring jako szansa na odnowę misji polskiego uniwersytetu*, w: Polska-Amerykańska Fundacja Wolności (Liderzy PAFW), W poszukiwaniu metody kształcenia liderów, Warszawa 2007, Stowarzyszenie Szkoła Liderów.

[3.] J. Axer, *Komandosi edukacji*, Rozmowa z prof. Jerzym Axerem opublikowana w dzienniku Gazeta Wyborcza, 2000/11/18-19.

Hubert Bożek

Pedagogical University of Cracow, PL

Logic as an Anti-platonic Antidote. A Case from Chwistek

Abstract. In my presentation I will propose an interpretation of Leon Chwistek's late semantic system known as 'Rational Semantics' or 'Rational Metamathematics' (RS, in short), which was fully outlined in his Limits of Science (1935). My objective is to demonstrate that RS not only could, but also should be used as an anti-metaphysical device – in particular as an anti-Platonic 'antidote' (in the traditional formulation of mathematical Platonism). The sketch of the argument to support my claim can be formulated as follows:

1. RS represents a coherent, anplatonic project of metamathematics.
2. The adoption of mathematical Platonism in foundations of mathematics results in a difficulty in differentiation between scientific/rational propositions and metaphysical speculatn in this context.
    - 2.1 Mathematical Platonism – in modern formulation – amounts to the following claim: 'one should adopt certain metaphysical statements (such as: «numbers exist»), whenever this becomes necessary for our best deductive theories to function'.
    - 2.2 The metaphysical propositions mentioned in 2.1. are adopted ad hoc. There is no strong (ex. logical) criteria for their adoption, except for those of purely practical nature, namely: theoretical functionality.
    - 2.3 A distinction between a rationally-adopted and a non-rationally-adopted proposition does not, as a whole, hold in mathematical Platonism, at least in the above shape and form.
3. In order to assure the rationality of deductive systems, we must either redefine mathematical Platonism or abandon it altogether.

In the final part of my paper I will argue that, depending on the interpretation, RS can be used to achieve either the former, or the latter.

Marianna Ciosek

PEDAGOGICAL UNIVERSITY OF CRACOW (RETIRED), PL

Stefan Turnau

UNIVERSITY OF RZESZÓW (RETIRED), PL

Harmonizing different kinds of mathematical thinking in mathematics education

Abstract. Classroom observation and research show that many – if not the majority of – students, when asked to decide on the truthfulness or falsity of a statement, answer (correctly or not) right away, without taking the time for reflection or investigation. Often a single example at hand is, to them, a sufficient foundation for their decision and its justification. An important aim of the teaching of mathematics should be the development of correct thinking habits in situations where truth is sought. Neither learning algorithms nor mathematical proofs could fulfill it. School mathematics is a good playground for it, but it would demand de-emphasizing elaborated proofs and harmonizing all kinds of inference: empiric, intuitive, through analogy, and deductive.

In the lecture we will show examples of correct and incorrect students' evaluations of sentences and their justifications, both collected as a result of our observation and research, as well as found in the literature.

Valéria Csépe

Research Centre of Natural Sciences of the Hungarian Academy of Sciences, HU

Neurocognitive development of magnitude processing

Abstract. One of the prerequisites of learning mathematics is the proper development of number sense. The concept of number sense used by researchers varies; meaning of numbers, number representation, the relationship or relative magnitude of numbers. The results of recent studies on the development of early mathematics contributed to the broadly accepted view that number sense is not a discrete set of skills and some of its components emerge early. It is quite clear that formal teaching of mathematics contributes to the changes due to the children's daily mathematical activities. Moreover, in many mathematics curricula the development of number sense is closely tied to problem solving, so that the numbers become central to make sense of a problem.

However, the development of basic skills is a result of concerted actions in the neurocognitive system. Studies focusing on the dynamics of neurocognitive development aim to follow the changes of the cortical network responsible for magnitude representation. Therefore, the concept of number sense does not only have different interpretations, it also has a different understanding in neuroscience. One of the most influential models called the 'triple code model' (see Dehaene and Cohen, 1998) gave impetus to intensive studies on the representations of numbers: visual code, verbal code and magnitude code. The results of our studies on magnitude processing shed light on the complexity of this seemingly simple skill. The presentation will focus on our recent data, showing that visual discrimination tasks relying on magnitude comparison have a particular load on the executive function network (EFN).

Paul Ernest

UNIVERSITY OF EXETER, GB

Challenging Myths about Mathematics

Abstract. In this talk I will question and challenge three ideas about mathematics: that mathematics is

1. a unique and unified subject,
2. absolute, universal and value-free,
3. an unqualified force for good.

Instead I claim there is no such thing as mathematics, as the term 'mathematics' does not refer to a single object.

I challenge the view that mathematics is absolute, universal and value-free, arguing that there are powerful and legitimate reasons for viewing mathematical knowledge, proofs and the objects of mathematics as human social constructions. From this perspective, it cannot be claimed that mathematics is ethics and values free.

Lastly I argue that mathematics does harm as well as good in shaping thought in an amoral or ethics-free way, it supports Instrumentalism and ethics-free governance (manifested in warfare, psychopathic corporations, human and environmental exploitation) and in sorting and labelling learners and citizens in modern society.

I conclude that to overcome such myths we need to teach the philosophy and especially the ethics of mathematics alongside mathematics itself.

Maria Gokieli

University of Warsaw, PL

Marcin Szpak

University of Warsaw, PL

Experiment and visualization in school mathematics

Abstract. We want to argue for teaching mathematics with computers, treated as tools for visualizing and experimenting. We will build upon our didactical experience related to:

- a few years of organizing a course of mathematics for a broad audience of school students at the University of Warsaw ("Matematyka dla Ciekawych Świata");
- courses at the Interdisciplinary Centre of Mathematical and Computational Modelling of the University of Warsaw;
- courses for teachers at the Cardinal Stefan Wyszyński University in Warsaw;
- teaching at the school and university levels in Poland and in France.

We will comment on the feedback from pupils, students and teachers. We think that software used by researchers on one hand, and common office packages on the other, can be successfully used in school didactics. We will give examples of using software like Mathematica, Matlab/Scilab, R, Excel in presenting mathematical notions, getting familiar with and as a proposition of integrated teaching of sciences. We will also present risks related to teaching with computer tools.

Agata Hoffmann
UNIWERSYTET WROCŁAWSKI, PL

Stanisław Dróżdż and mathematics

Abstract. As far as teaching practice in mathematics is concerned we use different methods, forms and tools, which enable pupils to better understand both knowledge and abilities. Stanisław Dróżdż (1939-2007) was a concrete poet, who in his work used not only words but also art. Poetry, art and mathematics seem to be far from each other, but – when I saw Dróżdż's works I noticed the opportunity to use them as original tools in teaching mathematics. I will show his chosen work and different ways of using them in maths education. Some of them could be used as illustrations of some concepts (for example in early maths education). Others could be starting points in discovering and exploring regularities. There are also some of them, which show us a path leading to mathematical problems.

Stanisław Dróżdż whilst creating his concept-shapes used similarity and difference of some situations. Searching for similarities and difference, using analogies and contrast make it also possible to build in pupils' minds mathematical concepts. Dróżdż by putting together concept and shape worked in an analogical way to the practice in mathematical education – while defining concept we give them the name and (very often) symbol, and we try to visualize them by giving a "shape". Using Stanisław Dróżdż's works, it is possible to build another bridge between theory and practice, particularly for pupils to whom maths is not a life-long passion.

Marek Janasz

Pedagogical University of Cracow, PL

Automated theorem proving for Euclidean geometry

Abstract. In this talk we will present a method of automated theorem proving for Euclidean geometry, known as the area method. This method, due to Shang-Ching Chou (Chou, 1994), is applied to constructive geometry statements, which are defined in the following way:
Definition 1. A constructive geometry statement is a list $S = (C_1, C_2, ..., C_m, G)$ where C_i, for $1 \leq i \leq m$, are elementary construction steps, and the conclusion of the statement, G is of the form $E_1 = E_2$, where E_1 and E_2 are polynomials in geometric quantities of the points introduced by the steps C_i. In each of C_i, the points used in the construction steps must be already introduced by the preceding construction steps [3, p. 499].

We present automated proofs of some celebrated theorems, including Thales' theorem (Euclid, *Elements*, VI. 2). Our proofs are generated by the program WinGCLC. Furthermore, we will address some questions concerning the educational aspects of automated theorem proving. References

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Jakub Jernajczyk

AKADEMIA SZTUK PIĘKNYCH IM. EUGENIUSZA GEPPERTA WE WROCŁAWIU, PL

The circle – mathematical inspirations in philosophy and art

Abstract. The presentation will concern the visual, mathematical and symbolic features of the circle – a figure which has played a particular role in the history of science, culture and art. We will take a closer look at the ancient methods of establishing a circle's area, which have displayed the irrationality and infinity it incorporates. We will look at the philosophical symbol of the perfect figure, which can be found in the thought of Parmenides, Plato, Nicolas of Cusa, Pascal and Quine, amongst others. We will pay particular attention to the circle as a metaphor for the development of human knowledge. The classic shape of this metaphor will be visualized along with its new, original version, presenting an idealistic model of knowledge.

Alina Kalinowska

Uniwersytet Warmińsko-Mazurski, PL

Day to day of creating mathematical concepts in early education

Abstract. Learning mathematical concepts by younger students is a daily process of the cognitive socialization of the mind. Essential for this process are, among other things: ways of constructing cognitive situations by the teacher, their verbal and non-verbal signals, the type of interaction they have with the students and the day to day class space. Early mathematics education creates a context for learning the opportunities to use the knowledge, depending on the way mathematical meanings function in the mind of the individual. Identifying the characteristics of everyday early mathematics education allows for a deeper understanding of school contexts of anchoring mathematical meanings in the minds of the youngest pupils, as well as the and interpretation of the features of these meanings.

Michalina Kasprzak

ADAM MICKIEWICZ UNIVERSITY, PL

Teaching mathematics creatively through theatre for preschool children and in early childhood education

Abstract. Mathematics is often perceived as one of the most difficult scientific disciplines, so education in this subject can be very problematic for children. It's been scientifically proven that creative education enhances the learning process. The artistic development of the child conduces a multi-subjectives education, (eg. German scientific research). However, many teachers have a problem with an attractively conducting classes for children in preschool and in early childhood education with regards to mathematics education. The theatrical form is one of the art forms which help with a multi-faceted development of children. In this report I would like to present proposals for mathematics classes conducted with theatrical techniques or with the use of theatre in general. In my report I would like to also inspire readers to look for and implement creativity into education.

Hyung Kim

University of Texas Pan American, US

1. *Developing mathematics teachers' pedagogical identity in the classroom context*

Abstract. Research demonstrates that there is a widening disjuncture between the practices that are encouraged by teacher education programs and what teachers do in the classroom, and that the teachers' selves (including their beliefs and knowledge) are major determinants of what the teachers do in the classroom. In order to understand how teachers' practices are shaped, this study explored how a mathematics teacher's pedagogical identity develops in the social context of his or her classroom interactions and what challenges teachers perceive in advancing their pedagogical identities. This study draws upon the dialogical approach, as formulated by Akkerman and Meijer, to consider a teacher's positioning of the "self" between the I-nature and the me-nature, as well as upon Grootenboer and Zevenbergen's ideas on classroom milieu formation and Simon's Hypothetical Learning Trajectory. Data were collected from four in-service high school teacher participants. The findings shed light on the nature of teacher variability, the process of mathematics teachers' pedagogical identity development and the contribution that positioning the "self" between the I-nature and the me-nature makes to pedagogical identity development.

Hyung Kim

UNIVERSITY OF TEXAS PAN AMERICAN, US

2. *Students' understanding of expected value of a random variable*

Abstract. This study explores the students' understanding of the notion of the expected value of a random variable. In particular, the study attempts to find out whether students understand the statistics notion (the expected value of a random variable) in relation to the underlying mathematics concepts and whether students develop misconceptions that are related to statistics symbols and language. The data were collected from eight students who were enrolled in a calculus-based junior-level university statistics course. The results suggest that many students develop misconceptions in their understanding of the statistics term "random" and the symbol $E(X)$, which are caused by linguistic and notational contamination. The study further suggests that while the majority of students are able to develop computational accuracy in arithmetic, enabling them to calculate the expected values of estimators, most students lack understanding of the mathematical definition of the expected value of a random variable, that is, the mathematical concepts that underlie the notion of the statistics concept.

Magdalena Kubat

Pedagogical University of Cracow, PL

An unusual polyhedron in students' mathematical education

Abstract. Line segments contained in the boundary of faces of a polyhedron and not containing other vertices, except their ends, are called the "edges" of this polyhedron (see [1]). In school mathematical education, a student has to deal with some examples of polyhedra, such as prisms and pyramids (see [2]). However, the concept of a polyhedron and the related definition of its edges, vertices and faces are not fully understood by students (see [3]). My presentation contains an example of a polyhedron and related research concerning the problem of how students perceived the notions of the edges and vertices of polyhedra.

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Anna Kucharzewska

PEDAGOGICAL UNIVERSITY OF CRACOW, PL

Artur Nyklewicz

PEDAGOGICAL UNIVERSITY OF CRACOW, PL

Does a mathematician need a university?

Abstract. Who can consider himself to be a mathematician? Can someone who graduated in mathematics call himself a mathematician? Do modern universities educate mathematicians, or just people with a master's degree in mathematics? What influence on the education of future masters of mathematics has the „current version" of the university?

Humboldt's university idea involved the search for truth, knowledge production, educating people in the process of solving problems and a right to choose the research areas... Is Humboldt's concept of the three principles of unity: knowledge; professors and students; and research and education, still valid today?

Is this the twilight of universities or just a temporary crisis? Actually, for hundreds of years, since the first communities of scholars and students began to form, there were always hints about a crisis of universities. Once, the hints concerned the elitism, „ossification", maladjustment to life, the decay of morals, the greed of professors and the debauchery of students. Today, however, the seemingly common access to knowledge and a lack of entrance examsleads to: the reduction in the level of education, weaker students, less active professors, and obtained diplomas being worth less than the paper they are written on.

The question about the future of universities, not only in the context of demographic problems, as well as technological and social challenges, is up to date throughout the world. Does a mathematician need a university with all these difficulties and issues? And if so, how should such a university look like?

Joanna Major

Pedagogical University of Cracow, PL

On graphic representation of a tangent line to a function

Abstract. Geometrically, the derivative is interpreted as follows. Let a curve $y = f(x)$ be given and let us draw a straight line through the points $[a, f(a)]$ and $[a+h, f(a+h)]$, h being a fixed positive value. This straight line is called a secant with regard to the given curve. It is easily seen that the difference quotient $g(h)$ is the tangent of the angle a_h between the secant directed by the increasing abscissae and the x-axis. The limiting position to which the secant tends as h tends to 0 will be considered as the position of the tangent (Kuratowski, 1962). But how do we show this *limiting position*?

In our talk we sketch the basic facts of nonstandard analysis, i.e. in the product $\mathbb{R}^{\mathbb{N}}$ we define an equivalence relation on $\mathbb{N}$ by using $(a_n) \equiv (b_n) \Leftrightarrow_{df} \{n \in \mathbb{N} : a_n = b_n\} \in \mathcal{F}$. The set of hyperreal $\mathbb{R}^*$ is the quotient set $\mathbb{R}^{\mathbb{N}}/_{\equiv}$. Algebraic operations on $\mathbb{R}^*$ are defined in a standard way. The structure $\mathfrak{R}^* = (\mathbb{R}^*, +, \cdot, 1^*, 0^*, \prec)$ Is a totally ordered, non-Archimedean field (see Goldblatt, 1998).

In the last part of our talk we sketch to get the limiting position of the secant (tangent to the graph of the function) in the structure $\mathfrak{R}^*$ without using limits.

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Volha Malinouskaya

UNIWERSYTET MARII CURIE-SKŁODOWSKIEJ, PL

Urszula Oszwa

UNIWERSYTET MARII CURIE-SKŁODOWSKIEJ, PL

Young mathematical talents in the Belorussian educational system

Abstract. Belorussian educational system belongs to the traditional systems considerably more than modern ones, however, some of its areas still seem to be highly effective. These include long-term systematic educational support for the remarkably gifted students. The main aim of the presentation is to show possibilities of multidirectional development of young mathematically gifted students on the background of the main ideas of general education in Belarus. Efficacy of this system is related to many factors, among others including culturally-political situation of gifted students in this country. It is worthy to analyse some of the elements of this system in the context of Mathematics Olympic rankings towards the use in Western countries what is old, proven and effective in Eastern Europe.

Jacobus Gideon Maree

University of Pretoria, ZA

Career Construction in the mathematics classroom: Using an integrated, qualitative + quantitative approach to enhance learners' sense of self

Abstract. Career Construction Counselling and Self-Construction Counselling aim to help learners script their career-life stories. This approach is suitable for exploring personal meanings and for helping learners deal with many of the problems involving meaning. The aim of the paper is to demonstrate the implementation of an integrated qualitative + quantitative approach in career counselling in combination with a parallel approach in mathematics to elicit and harness the learners' reflexivity, enhance their sense of self and, ultimately, enable them to participate more actively in their career and self-construction and in the writing of their emerging career-life stories. A brief case study will be discussed to demonstrate the approach. The results cannot be generalised, but it seems that drawing on an integrated qualitative+quantitative approach in mathematics classrooms holds much potential to enable the learners in a variety of contexts to improve their mathematical performance and, more importantly, help them to make and execute informed decisions about their career-life journeys.

Yuuki Mori

HOSEI, JP

Computational Complexity of College Math Eigenvalue Problems

Abstract. Providing students with suitable complex exercises is crucial to keeping them motivated and leading them to deeper understanding. Making such problems manually takes mathematics teachers' precious time which can otherwise be used to mentor students. Many software tools and services for automatic generation of math problems are found on the Web, but all of them provide only materials of high school level or below. In addition, no standardized methods are provided to evaluate and control the computational complexity of generated problems. The authors newly proposed a framework for evaluating computational complexity from learners' view, aiming to apply our framework to automatic generation of college math problems with controlled computational complexity. Our framework helps teachers to prepare teaching materials and thereby to save their time for the interaction with their students.

Manuela Moscucci

University of Siena, IT

About the relation between relationships and teaching and learning mathematics

Abstract. Mathematics education is a very rich field of research. Even if there is a such quality and quantity of valuable research, the difficulties that students encounter in learning mathematics at school are widespread. This not only causes discomfort in children and young people, but also causes frustration in mathematics teachers. Moreover, one has to consider the social damage that results from it. In fact, due to difficulties in mathematics in secondary school, many young people are deprived, firstly, of the opportunity to acquire skills in mathematics, and secondly, of the choice of pursuing degree programs that provide math courses.

The author has accepted, since the early 2000s, the invitation of eminent mathematics education scholars to address part of the researchers' efforts to overcome the disconnection between scientific research in mathematics education and mathematics teaching practice. The aim of this presentation is to submit an approach to the problem of difficulties in mathematics consistent with this objective. In particular, the author will deal with the issue of relationships both in mathematics education research, shown in various meanings, and in mathematics classroom practice. Moreover, an educational path, MBSA, will be presented, which is designed for the restructuring of the relationship of a person with mathematics, and which is widely used during both math courses, in every levels of education, and mathematics education courses, addressed to either future teachers of mathematics or ones already in service.

Bohdan Józef Naumienko
GLOBALONE, PL

The Mathematical Foundations of Integral Education

Abstract. The aim of the presentation is to discuss the general mathematical approach to integral education. Of course, as it is a conference address, it is more of an outline, as completeness here would be more troublesome than illumination. Generally, our point of view is that any mathematical model presented to the students should be based on a hidden or explicit form of the category theory which we will be able to demonstrate inside a specific interpretation of a dynamic system well known to the audience. The author's hope that this approach, strongly related to the language of the lecture as well as to the perception of the audience, would result in implementing integral teaching, ranging from letters and natural numbers to coordinate-free tensor analysis on differential manifolds and unitary operators in Banach and Hilbert Spaces.

Jarmila Novotná

Charles University in Prague, CZ

Developing a Culture of problem solving through heuristic strategies

Abstract. It is a universally accepted truth that problem solving forms the basis for successful mathematics education. Problem solving is an indicator of the state of comprehension of the concepts that pupils are taught. They help their solvers realize what former knowledge is applicable in a new situation, what role this knowledge plays in it, and which piece of knowledge turns out to be useless, or even erroneous, and thus becomes an obstacle to further development of mathematical knowledge and pupils' skills. In my talk, I will present the results of a three-year project, The development of a culture of solving mathematical problems in Czech schools (Czech Science Foundation project P407/12/1939) focusing on the use of heuristic strategies in problem solving. Heuristic strategies have been used in Polya's and Schoenfeld's understanding of the concept. The theoretical background of the research was Brousseau's Theory of Didactical Situations.

The use of heuristic strategies will be explored from two different perspectives: how heuristic strategies develop pupils' understanding of mathematics through using them, and how teachers change in consequence to giving their pupils the chance to use these strategies.

Eva Nováková

MASARYK UNIVERSITY, CZ

Prediction and self-evaluation of pupils in solving non-standard mathematical tasks

Abstract. The experience obtained from solving the tasks of the Mathematical Kangaroo in the category of primary school pupils category is a rich source of inspiration for educational practice. In our contribution we link the process of solving Kangaroo tasks to the more general issue of meta-cognition. We present the results of a research aimed at finding the level of prediction and of self-evaluation of performance by primary school pupils during the process of solving a set of non-standard Mathematical Kangaroo tasks.

Paweł Perekietka

Klaudyna Potocka Lyceum and Teachers Training Centre, PL

The contributions of computer science to mathematics education. Examples from outside curriculum

Abstract. The world is slowly changing from a continuous one to a discrete one in many important fields. However, the educational world seems to be lagged behind. Logical reasoning, algorithmic problem solving and discrete mathematics have received very little attention in curriculums and in the teachers' preparation.

There have been some outside curriculum efforts in many countries (e.g. the CS Unplugged project, the MathIS project, the Bebras contest), also in Poland (e.g. the Koala competition), to present concepts of computing mathematics to chidren and teens.

It should be seen as a supplement of curriculums, but also as a tool to transform school curriculums in the future.

Barbara Pieronkiewicz

PEDAGOGICAL UNIVERSITY OF CRACOW, PL

Affective transgression in learning mathematics

Abstract. In my presentation I will explain where the idea of 'mathematical transgressions' stemmed from.

The concept of psychological 'transgression' has been introduced by Polish psychologist Józef Kozielecki. From his point of view, a man is an expansive creature, who intentionally crosses over his own limitations, in order to become who he may be. During the first Mathematical Transgressions conference we have adopted this term into the field of mathematics education. Since then, we have heard about some interesting applications and transformations of this term, i.e. the notion of cognitive transgression, transgressions regarding various didactical problems, transgression as a substitute for the notion of the epistemological obstacle, etc. However, it is the affect domain, that gave birth to the idea of mathematical transgressions. For this reason, I would like to start with some fundamental concepts emerging from this research area, and subsequently, I will introduce the notion of affective transgression, and show how it refers to the problem of low achievement in learning mathematics.

Jerzy Pogonowski
Adam Mickiewicz University, PL

Paradox resolution as a didactic tool

Abstract. We share with the audience a few reflections concerning our Puzzles course, offered mainly to students of cognitive science at the Adam Mickiewicz University in Poznań, Poland. Contrary to the usual mathematical exercises, math puzzles are often connected with that which is unexpected, which contradicts our every-day experience. Thus, such puzzles are instructive, as far as a critical attitude towards informal intuitions is concerned. They teach us that we should be cautious in relying on intuitions, which are sometimes very illusory.

Observing the students' activity during our course, we have noticed that it is much more easier for them to acquire small, concise chunks of dissipated knowledge rather than to listen to lengthy expositions of entire theories seldom illustrated with examples.

The puzzles are divided into thematic groups, including such topics as: the Infinite, numbers and magnitudes, movement and change, shape and space, orderings, patterns and structures, algorithms and computation, probability, logic. Many of them are connected with paradoxes, i.e. results which seem counterintuitive but are nevertheless true, which can be shown by resolving the paradox in question.

We claim that paradox resolution is very instructive as far as the development of correct mathematical intuitions is concerned. Obviously, one should use several standard (normal, typical, natural) exercises in teaching mathematics – they doubtlessly serve as proper tools for stabilization of intuitions. However, to see clearly the limitations of our mathematical intuitions, we should also investigate the objects which – for several reasons – are called pathological in mathematics. Such objects eventually become domesticated, thus leading to new mathematical domains.

Agnieszka Proszewska

JAGIELLONIAN UNIVERSITY, PL

The Unreasonable Effectiveness of Mathematics or the Simple Efficiency of the Linguistic Model?

Abstract. 'The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve" wrote the great physicist Eugene Wigner in his famous article "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" in 1960. Speaking about the miracle, Wigner seems not to be exaggerating. Faced with the overwhelming success of the natural sciences, which nowadays benefit from experiment and mathematical deduction as procedures for testing the validity of hypotheses, one can not ignore this almost magical relationship.

For many years this deep connection between the physical world and mathematics has been leading both philosophers and scientists towards mathematical realism. But during the last decade, when people also started to discuss the linguistic character of mathematics – not as a part of a formalist philosophy but as an object of linguistic and semiotic analysis – Wigner's classic article received additional interesting comments.

What is really applied when we apply mathematics? In my presentation I will discuss not only the classic arguments for mathematical realism such as the Quine-Putnam Indispensability Argument or the famous Richard Hamming's response to Wigner's original paper and his idea of four "partial explanations" for this phenomenon but I will also present the interesting idea of Sundar Sarukkai – the philosopher and physicist who claims that the view of mathematics as the medium of language and models can help us understand the mechanisms for its unreasonably effective applicability.

Jerome Proulx

Université du Québec à Montréal, CA

Looking at students: from a medical/deficit view on mathematical knowledge toward possibilities of mathematical actions

Abstract. Most work in mathematics education (research and teaching) focuses on students' learning of mathematics. The (tacit) orientation taken is to look at that learning and create a conjecture on what the students know or don't know, and then making an attempt at finding ways of helping students to develop, or better understand, this mathematical knowledge. In this presentation I suggest that there are two major difficulties with this attitude for mathematics education. The first difficulty concerns the fact that mathematical knowledge is seen as a thing, something someone can grab onto, as if it existed by itself, independently. This view on mathematical knowledge leads to a second difficulty, which is that it offers a "deficit" view of learning, influenced by medical orientations (see Bélanger, 1991). Considering mathematical knowledge as an external thing "to know about" unfortunately leads to comparing students' mathematics with an allegedly external mathematics. With that orientation, students are always seen as lacking something, as needing more. They are always seen in deficit. I argue that this view is problematic for conceptualising about mathematical activity, and even ethically, and that a change is needed toward looking at what is made possible by students' actions and where it can lead to, rather than focusing on something that is supposedly missing. This advocacy requires transformations in our current paradigms, and I present the theoretical groundings from ethics, cognitive science, biology, mathematics and didactics to support this view, as well as discussing the methodological shifts that this imposes for analysing students' mathematics.

Miroslawa Sajka

PEGDAGOGICAL UNIVERSITY OF CRACOW, PL

Eyetracking as a new method for research on mathematics education

Abstract. We will discuss new opportunities to run research on mathematics education from the neurodidactical point of view, specifically by using eyetracking method by presenting exemplary results of the research with the application of eyetracking to examine the strategies and difficulties junior and senior high school students, university students and experts in the process of problem solving activities.

We will present some illustrative advantages of using eye-tracking method over conventional tests and discuss the way of analyzing psychophysiological data, such as pupilometry, to examine the self assessment of the level of problems' difficulty and cognitive load during problem solving.

We will show how eyetracking methodology can provide us with additional information and insight not available by using any other methods into the subject of strategies used to analyze mathematical texts and solving mathematical problems and subjective assessment of given problem level of difficulty.

Maria Samborska

Pedagogical University of Cracow, PL

Engaging underachieving students in mathematical thinking. An example of a supplementary math course for middle school students

Abstract. Students' low achievement in mathematics is a matter of many teachers' and researchers' concern. In Polish schools, the most common way to help those who struggle with mathematics is organising remedial classes, which are usually very similar to regular lessons. What is often missed in such an approach is the students' attitude towards mathematics, which is an important variable that affects learning and achievement.

In my talk, I will present an experimental supplementary math course for middle school students which was designed as an alternative to traditional remedial classes. The main goal was to create an environment in which students can enjoy mathematics and change their negative attitude towards this subject. I will present the results of a pilot study conducted in the academic year 2014/15 on a group of middle school students. I will show samples of pupils' work and comment on their performance, as well as the feedback received from them.

Bernard Sarrazy

UNIVERSITÉ DE BORDEAUX, FR

Contract, transgressions and creation. An attempt to clarify the paradoxes of the didactical relationship in mathematics education using a didactical and anthropological approach

Abstract. During the first part of the lecture, we will study, from a theoretical point of view, the issue of transgression as an expected response given by a pupil, but unrequired by the teacher. This phenomenon is the paradoxical result of the contract which forms during any didactical relationship: "this is what you have to know, and from now on, think for yourself to show that you are able to create new uses out of what you have been taught; in other words, act in accordance with what I have taught you, but don't obey me!". So, transgression will be considered as a necessary condition for learning mathematics (different from the use of techniques, algorithm, and rules) whose conditions of existence stand at the crossroads of determinations which are both didactical (with reference to "the paradox of devolution" as defined by Guy Brousseau in the theory of didactical situations) and anthropological (with reference to the concept of "use" in Wittgenstein's anthropology and to his famous rule-following paradox).

During the second part, we will base our argument on various research in order to underline:

a) the relevance and the interest of this theoretical approach in order to gain a better understanding of the reasons for pupils' and teachers' recurrent difficulties (for example, "you know the lesson, the teacher says, but you didn't understand it."), and the reasons why some of the means intended to regulate these difficulties fail, and

b) the role of "backgrounds" (in the Searlian meaning of the word), such as familial educational practices and the didactical and pedagogical cultures of school environments (which are linked to values, beliefs, epistemological and pedagogical conceptions of the teachers) in order to account for the appearance of interindividual differences concerning the relations with transgression, and clarify the ways we can go beyond the initial paradox.

In conclusion, we will promote the idea of a "normative transgression" to describe this phenomenon of the sudden appearance of new creations ("transgressive" dimension) which are expected by the teacher and lived by the pupil as a measured disobedience, for it is basically in accordance with the "account books" of mathematicians (normative dimension). This is probably where the fascinating and singular essence of mathematical activity stands, between logical constraints and the boundless openness of creative possibilities.

Alan Schoenfeld

UNIVERSITY OF CALIFORNIA AT BERKELEY, US

What makes for powerful classrooms, and how can we support teachers in creating them?

Abstract. Most of us think we know what "good teaching" is. The problem is that we don't – opinions about "good instruction" differ, although research clearly says certain things are important. My research-and-development goal has been to do some ground clearing: to lay out a straightforward way of characterizing classrooms that produce students who are powerful thinkers, to test that characterization empirically, and then to fashion forms of professional development that support teachers' growth in the things that count. I'll discuss progress along those lines. The talk will include a discussion of the "Teaching for Robust Understanding of Mathematics" (TRUmath) framework, a description of how it can be used for research on teaching, and also the characterization of various tools used to support effective teaching and for robust professional development.

Jeppe Skott
Linnaeus University, SE

The Goldilocks Principle revisited: Balancing "proving that" and "proving why" in mathematics teacher education

Abstract. Many teachers face difficulties with reasoning and proving, especially when they support their students' work with these mathematical practices. I outline the background to a planned development project in primary/lower secondary teacher education that seeks to alleviate these difficulties. I argue that the project needs to deal with reasoning and proving in problem contexts that are 'sufficiently close' both to the challenges teachers encounter in mathematics classrooms and to the practices of reasoning and proving in the discipline of mathematics. This is uncontentious, as much recent scholarship on mathematics teacher education argues for the need to balance school mathematics and academic mathematics. A more specific (and possibly more contentious) suggestion is that, in the case of mathematical reasoning, this means balancing "proving that" and "proving why" in ways that build on the mathematical complexities of tasks that are used in school mathematics. To make my argument I draw on a conceptual framework called Patterns of Participation (PoP). PoP views teachers' acts and meaning making as their (re-)engagement in other past and present practices in view of the interactions that unfold in the classroom rather than as their enactment of reified knowledge and beliefs. I use PoP-interpretations of classroom episodes to exemplify both the challenges teachers face when dealing with mathematical reasoning and the tasks that may be used in mathematics teacher education. However, my paper is not an empirical piece in the usual sense, but an empirically informed theoretical essay that outlines the background to the development project.

Anita Sondore

DAUGAVPILS UNIVERSITY, LV

Elfrīda Krastiņa

DAUGAVPILS UNIVERSITY, LV

The comprehension of elements of combinatorics in real-life situations for primary school students

Abstract. Competence in mathematics has been identified at EU level as one of the key competences which should be developed in primary school. One of the latest topics in the primary school mathematics curriculum in Latvia is related to elements of combinatorics and the probability theory. Teaching this topic is expected to result in developing the students' problem-solving abilities and practical application skills. The integrated approach helps to better understand the need to use mathematics in real life.

In the article, we will analyze mathematics curriculums and problems associated with the understanding of the use of combinatorial elements in different life situations of primary school children through the results of students' tests and a survey. In the article, we will give proposals for improvement of the mathematics curriculum in basic education (1st grade - 6th grade) and methodical recommendations for teachers for delivering different solving strategies in school, for example, guessing and checking, practical and graphical models, tables, graphs, games with dice. The national diagnostic tests of third, sixth and ninth grade (2012-2014) and the students' survey results will be used in this research.

Hannes Stoppel

University of Münster, DE

Development of students' beliefs in mathematical understanding in relation to mathematics and its applications

Abstract. The study investigates the development of relationships between epistemological beliefs and the perception of mathematics in the course of a year. The intervention consisted of three supplemental courses of 22 students of grade 12 and 13 in German high schools, two devoted to coding theory and cryptography, and one to the mathematical aspects of cosmology and particle physics.

During the first quarter of the school year, the students studied the mathematical foundations of the course topics. In each of the last three quarters, the students were offered a choice between several project topics, or could find a topic themselves. They worked on the topics alone or in pairs. At the end of each quarter, the students had to present their results.

At the end of the second and fourth quarter, the students were interviewed, following a semi-structured concept, for about 25 minutes. The first interviews showed that the students' opinions about mathematical understanding were related to their definition of mathematics and their choice of project topics. By dividing mathematical understanding into active and passive categories, it became obvious that the students who used abstract attributes defined mathematics as suitable for the usage of applications and chose an abstract topic for projects, and vice versa. As it became evident in the latter interviews, most students' understanding of mathematics changed when their definition of mathematics and their choice of topics were done both from an application and an abstract point of view during the third and fourth quarter of the year.

Ewa Swoboda

UNIVERSITY OF RZESZÓW, PL

"Transgressions" of a didactical problem concerning geometric reasoning. From the researcher's workshop

Abstract. Didactics of mathematics as a scientific discipline is transgressive – it is rooted in mathematics, though it exceeds the cognitive limitations related to the practice of mathematics itself. It enters into other areas of research and interprets one's results from different perspectives.

In my presentation I will show examples from my own research – how, while building a theory concerning teaching geometry, I support my findings with data from other scientific areas. I will show how findings from history and philosophy of mathematics, neuroscience, physics and psychology affect my explanations of the phenomena observed during the study of dynamic geometric reasoning.

Maciej M. Sysło
University of Wrocław, PL

Learning Mathematics Supported by Computational Thinking

Abstract. In this presentation, we focus on applying computational thinking mental tools to specific topics in school mathematics. Our goal is twofold. On the one hand, we suggest how to extend and enrich traditional topics in school mathematics by applying computational thinking and, as a result, obtain solutions which use and are supported by the power of computer science as a discipline, as well as computers as computing tools. On the other hand, our approach to dealing with topics in mathematics using computational thinking and computing tools contributes to constructionist learning, which is learning by doing and making meaningful objects in the real world – here, computer solutions in computing environments. The mental tools used herein, dealing with specific topics in mathematics, include: number representations, reductive thinking, approximation of numerical and intractable problems, recursive and logarithmic thinking, heuristics.

Paola Vighi

UNIVERSITY OF PARMA, IT

Abstract paintings, objects and actions: how to promote geometrical understanding

Abstract. The paper presents the description and the analysis of an activity carried out in kindergarten, with children who are 5 years old. The starting point is a painting of Wassily Kandinsky, titled "Soft Hard", and its reproduction made by the pupils, following particular tasks prepared by the teacher. Different pedagogical and psychological studies about visual perception deal with laws of visual data organisation. They show that visual perception may hinder the ways of seeing figures; in other words, young pupils observe certain figures rather than others in a picture. With reference to the 4-8 years old children's reading of images, which are present in a painting, some pedagogical studies show that in presence of abstract art paintings, pupils show a "referential need" to identify a likeness, to find out which objects there are in a painting. Starting from this assumption, in collaboration with a kindergarten teacher, I planned an activity based on the copy of each geometrical shape present in the Kandinsky painting chosen as well as its reproduction, by gluing the shapes on a sheet of paper. From a mathematical point of view, this activity involves not only concepts such as "top and bottom" or "forward or back", but also symmetry and geometrical transformations as rotations and similarities, as well as the mutual positions of geometrical shapes on the space of the paper. The analysis of the results produces interesting information about children's approach to geometrical understanding.

Shlomo Vinner

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Scientific thinking versus religious thinking from a view point of a secular science educator

Abstract. In my talk I'll characterize Science Education as a research discipline. I will characterize scientific thinking versus religious thinking. The main difference is that scientific truths are refutable, whereas religious truths are irrefutable. Thus, relying on Popper, we can consider religious thinking as pseudo-scientific thinking. The talk will compare the contributions that science and religion can offer to face the main problems of human beings: mortality and suffering.

Science can hardly facilitate our dealing with these problems. On the other hand, the solutions which religion offers us have their own problematics.

Barbara Wawrzacz

PRIMARY SCHOOL IN STARE BIELICE, PL

The use of building blocks in construction of squares and cubes representing the numbers – transition from the concrete world to the world of operational reasoning

Abstract. Didactic classes in a pupil-friendly school require its transformation from a teaching environment to a learning environment, in which, apart from the implementation of new educational concepts oriented towards innovative thinking, as well as the development of talents and social competences, the creation of associations, deductive and inductive reasoning, it is important for the teaching "teacher-guide" to create a friendly learning environment and strengthen the motivation of pupils. The important elements of such an environment are: the sense of perpetration in pupils, and triggering a desire to cooperate and provoke reflection. In this paper, two types of information were discussed, i.e. the contents and products which the pupils used during an algebra lesson (topic: squares and cubes of numbers) through:

- cognition, memorisation, appraisal, convergent and divergent thinking;
- selection between figurative, symbolic and/or semantic contents engaging them into their preferred style of learning;
- stimulation of creativity and reflexive thinking about their own learning.

To fortify the "external motivation" of pupils, the lesson was carried out by an application of the exercise method, using building blocks for this purpose, and mobilizing the pupils' cognitive processes to assimilate the discussed mathematical problems through play. The pupils' motivation for creative activity and the willingness to take up effort of reasoning via analogy has been presented. On the other hand, the role of the teacher-guide following the pupil's cognitive processes through asking open-ended questions was discussed.

Particular strengthening of information coding in pupils' memory tracks due to information imaging and association of reasoning mobilization with readiness to repeat the exercise (play) has been indicated in the paper. Weight of independent acquisition of knowledge by action and

finding sense in such action, consequently, independent reasoning, has been underlined.

During work presentation pieces of the lesson recorded on an electronic medium will be displayed.

Erich Ch. Wittmann

TECHNISCHE UNIVERSITÄT DORTMUND, DE

Structure-genetic didactical analyses – empirical research of the first kind

Abstract. In mathematics education, theories of teaching and learning based on disciplines different from mathematics ("imported" theories) are widely dominating the field.

This imbalance greatly reduces the impact of mathematics education both on teacher education and on the teaching practice.

In order to return to a balanced situation it is necessary to pay more attention to theories which are based on mathematics. As an example of such a "homegrown" theory, the talk will present the structure-genetic didactical analysis, the research method of mathematics education conceived of as a "design science".

Lidia Zaręba

Pegdagogical University of Cracow, PL

The role of visualization in the process of generalization

Abstract. In my presentation, I discuss the results of individual observations of pupils aged 13-14. These children undertook an attempt to solve problems which were designed in order to bring about an inductive type of generalising.

The main aim of the study was to classify typical methods of proceeding, which represent the pupils' ways of reasoning and to determine what influence a particular method of proceeding may have on the final outcome of their work.

These results indicate the general pupils' strategies of generalising. Presumably, visual thinking produces a positive effect on the pupils' process of generalizing and abilities to describe regularity with the aid of a letter symbol.

Waclaw Zawadowski

UNIWERSTET PRZYRODNICZO-HUMANISTYCZNY W SIEDLCACH, PL

How to use a pocket calculator for developing and sustaining readiness in mental calculation

Abstract. A method will be presented on how to use simple calculators to enhance mental calculations and the estimation of arithmetical operations which are useful in everyday life.

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